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Theory of premixed-flame propagation in large-scale turbulence

By P. CLAVIN AND F. A. WILLIAMS†

Laboratoire de Dynamique et Thermophysique des Fluides, Université de Provence-Centre Saint Jerome, 13397 Marseille, France

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A statistical theory is developed for the structure and propagation velocity of premixed flames in turbulent flows with scales large compared with the laminar flame thickness. The analysis, free of usual closure assumptions, involves a regular perturbation for small values of the ratio of laminar flame thickness to turbulence scale, termed the scale ratio ϵ, and a singular perturbation for large values of the non-dimensional activation temperature β. Any effects of the flame on the flow are considered to be given. In this initial study, molecular coefficients for diffusion of heat and reactants are set equal. The results identify convective–diffusive and reactive–diffusive zones in the flame and predict thickening of the flame by turbulence through streamwise displacement of the reactive–diffusive zone. Profiles for intensities of temperature fluctuations and for streamwise turbulent transport are obtained. A fundamental quantity occurring in the analysis is the longitudinal displacement of the reactive–diffusive zone in an Eulerian frame by turbulent fluctuations, and to first order in the scale ratio this equals the longitudinal displacement of fluid elements in an Eulerian frame by turbulent fluctuations, herein termed simply the Eulerian displacement. To first order in the scale ratio it is found that, if the Eulerian displacement experiences the same type of statistical non-stationarity as the corresponding Lagrangian displacement, then the diffusion approximation is valid for streamwise turbulent transport but the turbulent flame thickens as time increases, while if the Eulerian displacement is statistically stationary then the diffusion approximation necessitates a negative coefficient of diffusion in part of the flame but the flame thickness remains constant. By carrying the analysis to second order in the scale ratio it is shown that the turbulent-flame speed exceeds the laminar-flame speed by an amount proportional to the mean square of the transverse gradient of the Eulerian displacement. This result can be understood from the mechanistic viewpoint of a wrinkled laminar flame in terms of the increase in flame area produced by turbulence. Thus the theory provides a precise statistical quantification of the model of the wrinkled laminar flame for describing structures of turbulent flames.

1. Introduction

Difficulties in developing theoretical descriptions of turbulent flames are so great that in most work *ad hoc* approximations, termed 'modelling assumptions', are introduced to render calculations tractable. The predictions that result always are question-

† Permanent address: Department of Applied Mechanics and Engineering Sciences, University of California, La Jolla, California 92903.

able because they rely on untested assumptions. An alternative to this mainstream in research on turbulent combustion is to study limiting cases in which perturbation methods may be employed to provide theories independent of modelling assumptions. A step in this direction has been reported previously in work in which the turbulence intensity was treated as a small parameter and the overall activation energy as large (Williams 1975). In the present paper a somewhat analogous perturbation theory is developed, not beginning with the small-intensity approximation but instead making use of the assumption that the length scales of all fluctuations in the spectrum of turbulence are large compared with the thickness of a laminar flame. The analysis is completed, without introduction of arbitrary modelling approximations concerning closure, to yield predictions for the flame structure and flame speed.

The configuration considered is the same as that of Williams (1970). A steady one-dimensional flow of a premixed combustible passes at low Mach number through a turbulence-producing grid oriented perpendicular to the flow direction. A planar turbulent flame, extending normal to the flow, is established downstream from the grid at a sufficiently large distance to prevent flame holding by individual elements of the grid. The mean flow velocity is adjusted to keep the flame position stationary. Experimentally, a gradual divergence of the duct may aid in maintaining a fixed flame. The mean velocity of the flow just ahead of the flame is the turbulent-flame speed. The theoretical results may be expected to apply more generally to open, planar, non-accelerating, turbulent flames in flows with negligible values of the mean rate of strain.

The approximation of large-scale turbulence is applicable in particular under conditions for which the turbulent flame consists of a wrinkled laminar flame. From the results of the present development it will be found that the dominant physical phenomenon analysed is flame wrinkling. There is a common misconception that if a turbulent flame consists of a wrinkled laminar flame then its structure is entirely understood. This idea is false because the geometry of the wrinkling in turbulent flows has not been studied thoroughly. For example, theories that predict turbulent-flame speeds from the wrinkled-flame model introduce arbitrary assumptions concerning the flame shape (Williams 1965). The principal contribution of the present work may be viewed as a precise formulation of kinematic aspects of flame wrinkling and of the consequent influences on the structure and speed of turbulent flames.

To aid analysis of the flame wrinkling the formalism is developed by use of a coordinate system that moves in the longitudinal direction with the fluctuating velocity of a flamelet. Since the laminar flame is thin, in an Eulerian frame large fluctuations in temperature may occur even if flame displacements are small compared with turbulence scales. However, these large fluctuations do not occur in the moving frame adopted. This simplifies the analysis and extends its validity to turbulence of higher intensity than would otherwise be acceptable. In fact, the approximation of low turbulence intensity, which restricted the previous analysis (Williams 1975) to grid turbulence in the final stage of decay, is not used in the present theory.

In statistical theories of wrinkled flames concern arises about instabilities of the planar laminar flame. Hydrodynamic instabilities (Landau 1944) and thermal–diffusive instabilities (Sivashinsky 1977*a*) leading to cellular-flame phenomena of various types (Markstein 1964) are known to exist, and the character of these instabilities has begun to be clarified (e.g. Sivashinsky 1977*a*, *b*; Joulin & Clavin 1978).

The present work constitutes a first step in which attention is focused on the effects of upstream velocity fluctuations when the laminar flame is stable. Therefore restrictions are introduced to assure that the forcing perturbations, imposed externally, cannot trigger self-evolution of a non-planar or unsteady structure inherent in the flame. Specifically, the Lewis number (the ratio of thermal to molecular diffusivity) is set equal to unity, and the density change associated with the heat release of combustion is neglected.

The value selected for the Lewis number excludes the thermal–diffusive instabilities, which are known to occur only if the Lewis number is less than a critical value which is slightly below unity or greater than a critical value appreciably above unity (Sivashinsky 1977*a*, *b*; Joulin & Clavin 1978). This restriction limits the present analysis to smoothly wrinkled flames and leaves turbulent flames of cellular types for future studies. It is of interest to extend the theory to combustibles with Lewis numbers different from unity and to mixtures containing reactants with differing coefficients of diffusion. This necessitates working with different conservation equations for energy and species, instead of the single equation treated herein. It should be observed that, although the present formulation is developed for only a one-reactant system, it is applicable to a two-reactant system in which either both reactants have equal molecular diffusivities or the deficient reactant is sufficiently dilute; in the latter case, applicable either if there is an excess of inert components and the mixture is far from stoichiometry or if there is a negligible concentration of inert components, the relevant molecular diffusivity is that of the limiting component. The restriction to one-reactant systems with a Lewis number of unity is introduced here for the purpose of exhibiting the main features of the gradient-expansion method in the simplest manner possible. In a sequel to this paper we extend the present analysis to consider a Lewis number differing from unity by an amount of the order of the reciprocal of the non-dimensional activation temperature. This extension shows that, when the Lewis number is above the lower critical value for instability, the results differ from those derived here only through the effect of the fluctuation on the maximum value of the temperature, which then differs from the adiabatic flame temperature. Additional influences on flame speed and flame structure occur, but the phenomena derived herein remain. In the unstable situation the analysis would be appreciably more difficult but also would provide new and interesting features concerning turbulent cellular flames.

The neglect of density changes associated with heat release circumvents completely the Landau instability. Although the introduction of this crude approximation raises questions concerning the degree of generality of the results, it is possible that the theory will continue to hold when the density changes are sufficiently small for the Landau instability to be negligible over the time scale of the upstream fluctuating field or over the time scale of observation. To test this hypothesis it would be of interest to extend the theory to consider density changes across the flame. Difficulties in observing Landau instability suggest that the present results may be valid for appreciably large density ratios. In particular, this instability has seldom been encountered for laminar flames. The experimental observation of the existence of wrinkled laminar flames in turbulent flows under suitable conditions (Libby & Williams 1976) lends support to the contention that the present theory applies in some respects to many real flames.

Fluctuations in temperature and in reactant concentration in the approach flow are neglected in the following analysis. This corresponds to the usual experimental situation, in which a uniform isothermal mixture of reactants is passed through a non-heated grid. Fluctuations in temperature, for example, can be introduced by heating the grid. In general, such upstream fluctuations in temperature or concentration necessitate retaining at least two conservation equations in the analysis. We have not investigated this problem, although fundamental difficulties may not arise because instability phenomena are not anticiplated. We have included upstream temperature fluctuations in the one-equation model. This is consistent with the full equations if temperature fluctuations are perfectly correlated with concentration fluctuations and if the relative amplitudes of temperature and concentration fluctuations are such that the sum of the thermal and chemical enthalpies remains constant. The analysis will not be included here because of the excessively complicated algebra. The results influence some details of the structure of the turbulent flame, but whether they produce a flame-speed correction dependent on the turbulence scales can be ascertained only by carrying the analysis beyond second order in the ratio of length scales.

The problem will be formulated in the following section, where the transformation to the moving co-ordinate system is made. Analyses to first and second orders in the ratio of length scales appear in the subsequent section. The second-order theory is needed in order to obtain a non-vanishing constant correction to the flame speed. Both of these analyses involve matched asymptotic expansions for large values of the non-dimensional activation temperature, there being separate convective–diffusive and reactive–diffusive zones in this limit, just as for the ordinary laminar flame (Williams 1971). The results for flame speed and for flame structure are presented in subsequent sections.

2. Formulation

For simplicity, consider an exothermic reaction of the type $A \rightarrow B$, and assume that A and B have constant and equal specific heats c_p. If the pressure is constant and if the Lewis number is unity, then an exact solution to the equation for the sum of the thermal and chemical enthalpies is $c_p T + Q_A Y_A = c_p T_f$, where T is the temperature, Y_A denotes the mass fraction of species A, Q_A represents the heat released per unit mass of species A consumed, and T_f is the adiabatic flame temperature. Use of this result enables the equation for conservation of thermal enthalpy to be written as

$$\frac{\partial T}{\partial \hat{t}} + \hat{\mathbf{v}}\,.\,\hat{\nabla} T - \hat{\nabla}\,.\left(\frac{\lambda}{\rho c_p}\hat{\nabla} T\right) = B(T_f - T)^n \exp(-T_a/T), \tag{1}$$

where $\hat{\mathbf{v}}$ represents the velocity vector and λ the coefficient of thermal conductivity. The density ρ of the gas mixture has been assumed to be constant. In the term for chemical production of heat, T_a denotes the constant activation temperature for the Arrhenius reaction (the ratio of the activation energy to the universal gas constant), n is the order of the reaction with respect to the reactant A, and the constant prefactor B is the product of $(c_p/Q_A)^{n-1}/\rho$ with the frequency factor for the reaction rate. Hats on time and gradients indicate dimensional time and space co-ordinates.

Non-dimensional co-ordinates are introduced according to

$$(x, y, z) = (\hat{x}, \hat{y}, \hat{z})\, u_L \rho c_p/\lambda \quad \text{and} \quad t = \hat{t} u_L^2 \rho c_p/\lambda,$$

where u_L is the laminar-flame velocity while $d = \lambda/(u_L \rho c_p)$ and $t_L = d/u_L$ are the laminar-flame thickness and transit time, respectively. The mean flow is in the $\hat{x}$ direction, $\hat{y}$ and $\hat{z}$ being transverse co-ordinates. The non-dimensional vector of velocity fluctuations is denoted by $\mathbf{v} = (u, v, w) = (\hat{u} - u_T, \hat{v}, \hat{w})/u_L$, its ensemble average being $\bar{\mathbf{v}} = 0$. Here u_T is the average velocity of the flow, the turbulent-flame speed. In the statistical sense v is taken to be stationary in t and homogeneous in the y and z directions.

In terms of the dimensionless normalized temperature or concentration variable $Y = (T_f - T)/(T_f - T_0)$, where T_0 is the upstream temperature, (1) becomes

$$\partial Y/\partial t + m\, \partial Y/\partial x + \mathbf{v}.\nabla Y - \nabla^2 Y = -\Lambda_L F(Y), \tag{2}$$

where

$$m = u_T/u_L, \quad F(Y) = \beta^{n+1} Y^n \exp[-\beta Y/(1-\alpha Y)], \tag{3}$$

$$\alpha = (T_f - T_0)/T_f, \quad \beta = T_a(T_f - T_0)/T_f^2$$

and

$$\Lambda_L = [\lambda B \exp(-T_a/T_f)]/[T_a^{n+1} T_f^{-2n} \rho c_p u_L^2 (1 - T_0/T_f)^2]. \tag{4}$$

Here α is a non-dimensional measure of the heat release, β is the appropriately non-dimensionalized activation temperature and Λ_L is the conventional eigenvalue for the propagation velocity of the laminar flame. It is anticipated that (2) will possess acceptable solutions only for a particular value of m, which then may be thought of as a burning-rate eigenvalue specifying the turbulent flame speed u_T. Note that, with density constant, overall mass conservation adds the requirement $\nabla . \mathbf{v} = 0$ to (2). The only additional conditions needed are the boundary conditions that Y approaches unity as x approaches $-\infty$ and Y approaches zero as x approaches $+\infty$.

Analysis of (2) provides us not only with a value of u_T but also with the time and space dependence of Y in terms of the fluctuating part of the velocity field. In this sense the fluctuating velocity field is considered to be a given quantity, known in advance. This velocity field may be altered from its upstream value by the combustion process in crossing the flame. If that occurs, then the fluctuating velocity field appearing in the solution of (2) must include this modification. Since our objective here is only to investigate the influence of the turbulence on the combustion, consideration is not given to the Navier–Stokes equations, which are needed to find the modification to the turbulence by combustion. In future analysis it may be possible to use the expression for Y in terms of $\mathbf{v}$, obtained here, to find this modification of $\mathbf{v}$ by combustion.

The quantity α, which obeys $0 < \alpha < 1$, typically is a parameter of order unity. If pressure and the average molecular weight remain constant, then according to the ideal-gas law the change in density across the flame is $\rho_f - \rho_0 = -\rho_0 \alpha$. Therefore the approximation of constant density corresponds to $\alpha \ll 1$. For the laminar problem it is known that the eigenvalue Λ_L is independent of arbitrarily imposed conditions at the upstream boundary only within the context of an asymptotic expansion for $\beta \gg 1$ (Williams 1971). Although $\beta = \alpha T_a/T_f$, the conditions $\beta \gg 1$ and $\alpha \ll 1$ are mutually consistent for sufficiently large activation temperatures. Here we investigate (2) in terms of an asymptotic expansion for large values of β. The essential feature is that $F(Y)$ is a non-negative, single-peaked function that is zero when $Y = 0$, that becomes negligibly small as Y approaches unity and that depends parametrically on β in such a way that its peak becomes increasingly narrow and moves progressively closer to $Y = 0$, with its magnitude continually decreasing for values of Y appreciably greater than zero, as β approaches infinity. The specific functional form appearing in (3) is inessential.

If the fluctuating velocity field $\mathbf{v}$ varies weakly over the flame thickness as anticipated for flames in large-scale turbulence, then it may be assumed that the local structure of the flame remains close to laminar at each time and, for high values of β, possesses a thin, inner, reactive zone. Let $x = \alpha(y, z, t)$ be the equation of this reactive 'surface' in non-dimensional co-ordinates. This unknown, randomly moving surface must be considered to be an eigenvalue which is a function of space and time, to be determined as m, with successively improved accuracy at each step of the perturbation analysis presented below. The method provides directly an expression for the flame-front fluctuations in terms of the fluctuating velocity field, a result which is interesting because it may be compared with experiment relatively easily. It is assumed that the time average of α vanishes, the mean velocity of the flow being the turbulent flame speed.

When analysing flames in large-scale turbulence it is convenient to work in a co-ordinate system that moves in the longitudinal direction with the unknown, local, fluctuating position of the laminar-flame front. In terms of the variables $\xi = x-\alpha$, $\eta = y$, $\zeta = z$ and $\tau = t$, (2) becomes

$$\frac{\partial \Psi}{\partial \tau} + (m-\alpha_t+u-\mathbf{v}\,.\,\nabla\alpha+\nabla^2\alpha)\frac{\partial \Psi}{\partial \xi}+v\frac{\partial \Psi}{\partial \eta}+w\frac{\partial \Psi}{\partial \zeta}$$
$$= (1+|\nabla\alpha|^2)\frac{\partial^2 \Psi}{\partial \xi^2}-2\alpha_y\frac{\partial^2 \Psi}{\partial \xi\,\partial \eta}-2\alpha_z\frac{\partial^2 \Psi}{\partial \xi\,\partial \zeta}+\frac{\partial^2 \Psi}{\partial \eta^2}+\frac{\partial^2 \Psi}{\partial \zeta^2}-\Lambda_L F(\Psi), \quad (5)$$

where $\Psi(\xi,\eta,\zeta,\tau) = Y(x,y,z,t)$, subscripts on α denoting partial derivatives of the function $\alpha(y,z,t)$. Note that if there is no spatial variation of α then there exists a trivial solution to (5), viz.

$$\Psi = \Psi_L(x-\alpha), \quad m = 1, \quad \alpha_t = u, \quad (6)$$

where the subscript L refers to the laminar solution. This solution corresponds to a longitudinal motion of the laminar profiles, random in time and uniform in transverse co-ordinates.

Any change in this physical behaviour must be attributable to transverse spatial variations of α, which are related, as suggested by (6), to spatial variations of the longitudinal fluctuating velocity u. Therefore an expansion in gradients of u about the randomly moving laminar solution (6) can be considered. Since the flame thickness d has been taken as the scale for non-dimensionalizing the lengths, the small parameter ϵ of this expansion is the ratio of the laminar flame thickness to the spatial scale over which u varies. As in the laminar case, an asymptotic expansion for large values of β must also be introduced in order to treat the nonlinear production term $F(\Psi)$.

3. Analysis

The solution to (5) possesses an outer zone (ξ and Ψ of order unity) where the reaction is negligible and an inner region ($\beta\xi$ and $\beta\Psi$ of order unity) where convection is negligible in the first approximation. The corresponding analysis for the laminar flame may be found in the work of Williams (1975), for example, who shows that the outer solution is $\Psi_L(\xi) = (1-e^{\xi})H(-\xi)$, where $H(\xi)$ denotes the unit step-function.

With the notation $\sigma = \beta\xi$ and $\psi = \beta\Psi$, the inner solution may be expressed as a series, $\psi_L(\sigma) = \psi_0(\sigma) + \beta^{-1}\psi_1(\sigma) + \ldots$; the functions $\sigma(\psi_0)$ and $\psi_1(\psi_0)$ are written out by Williams (1975). The expansion for the laminar eigenvalue is $\Lambda_L = \Lambda_0 + \beta^{-1}\Lambda_1 + \ldots$, where $\Lambda_0 = (2n!)^{-1}$ and $\Lambda_1 = 2\Lambda_0[\alpha(n+1)(n+2)/2 - \gamma]$, with

$$\gamma = \int_0^\infty [1 - \Gamma(x)]\,dx, \quad \text{in which} \quad \Gamma(x) = \left[\int_0^x f(y)\,dy/n!\right]^{\frac{1}{2}}.$$

Here the function $f(x) = x^n e^{-x}$ is the first approximation in the inner variable to $\beta^{-1}F(Y)$ in (3). In the approach adopted here, the asymptotic expansion of this type in β^{-1} is performed first and the gradient expansion, in ϵ, afterwards. Moreover, indices related to the expansion in β^{-1} will not be introduced, since only the dominant term in the expansion is needed; in the following Ψ, m, Λ_L and α represent quantities of lowest order in the asymptotic expansion in β^{-1}.

By formally introducing the stretched variable σ into (5), it may be seen that, in the reaction zone (the inner region), to lowest order in β^{-1} the equation

$$(1 + |\nabla\alpha|^2)\,\partial^2\Psi/\partial\xi^2 = \Lambda_L F(\Psi) \tag{7}$$

applies, provided that the characteristic lengths and times for turbulence are larger than those of the laminar flame. Except for the term $|\nabla\alpha|^2$, (7) is identical to the corresponding inner equation of the laminar flame. The same method of solution may therefore be employed. The downstream boundary condition, obtained by matching to an outer zone downstream, is $\Psi = 0$ at $\xi = \infty$, and therefore a first integral of (7) is

$$\left(\frac{\partial\Psi}{\partial\xi}\right)^2 = \frac{\frac{1}{2}\Lambda_L}{(1 + |\nabla\alpha|^2)}\int_0^\Psi F(\Psi)\,d\Psi. \tag{8}$$

From the results quoted above, to lowest order for the laminar flame,

$$\tfrac{1}{2}\Lambda_L \int_0^\infty F(\Psi)\,d\Psi = 1,$$

which may be used, in matching of the inner solution given by (8), to provide the boundary condition

$$[\partial\Psi/\partial\xi]_{\xi=0-} = -(1 + |\nabla\alpha|^2)^{-\frac{1}{2}} \tag{9}$$

for the slope of the outer solution in the upstream region. In addition, the boundary condition $\Psi(0, \eta, \zeta, \tau) = 0$ for the outer solution in the upstream region is obtained through matching, while the boundary condition $\Psi(-\infty, \eta, \zeta, \tau) = 1$ follows from the conditions specified upstream. The problem that remains is linear, viz. that of solving (5) in the upstream region with $F = 0$ and with the boundary conditions just stated. The flame-speed parameter m and the fluctuating position α of the flame front are obtained by requiring the solution of the second-order equation to satisfy three boundary conditions.

Here this linear problem is solved by a perturbation technique using a regular expansion in powers of the gradients of $\mathbf{v}$. This expansion, justified by the small-ϵ approximation, does not exclude influences of the flame on the flow, but if such influences occur then they must not be sharp enough to violate the small-gradient expansion. Furthermore, such influences would cause the analysis not to be closed, in the sense that properties of $\mathbf{v}$ cannot be known without further consideration of the flame. Let $m = m_0 + m_1 + m_2 + \ldots$, $\alpha = \alpha_0 + \alpha_1 + \alpha_2 + \ldots$ and $\Psi = \Psi_0 + \Psi_1 + \Psi_2 + \ldots$ denote the expansions in ϵ, where subscripts identify the power of the gradient. The expansion of $\mathbf{v}$ becomes

$$\mathbf{v}(x,y,z,t) = \mathbf{v} + (\xi + \alpha_0)\,\mathbf{v}_x + \alpha_1 \mathbf{v}_x + \tfrac{1}{2}(\xi + \alpha_0)^2\,\mathbf{v}_{xx} + \ldots, \tag{10}$$

where $\mathbf{v}$ on the right-hand side denotes the value $\mathbf{v}(0,y,z,t)$ of the fluctuating velocity field at the point $x = 0$, defined as the average position of the reactive–diffusive zone. Similar notation is used in the following for $\mathbf{v}$, $\mathbf{v}_x$, etc.

Putting these expansions into (5), we obtain to lowest order in gradients for the outer equation

$$\frac{\partial \Psi_0}{\partial \tau} + (m_0 - \alpha_{0t} + u)\frac{\partial \Psi_0}{\partial \xi} - \frac{\partial^2 \Psi_0}{\partial \xi^2} = 0. \tag{11}$$

As stated previously, the solution to (11) satisfying the boundary conditions is

$$\Psi_0 = \Psi_L(\xi) = 1 - \exp \xi, \tag{12}$$

with $m_0 - \alpha_{0t} + u = 1$. Since α is the fluctuating part of the flame position, the definitions yield for the time average $\overline{\alpha}_0 = 0$, whence $\overline{\alpha}_{0t} = 0$. In view of the fact that $\overline{u} = 0$, we then obtain $m_0 = 1$ and $\alpha_{0t} = u$, where, according to our notation, u denotes $u(0,y,z,t)$. Overbars will always indicate time averages taken at fixed values of the spatial co-ordinates of the function concerned.

At first order the equation and boundary conditions are

$$\left.\begin{aligned} &\frac{\partial \Psi_1}{\partial \tau} + \frac{\partial \Psi_1}{\partial \xi} + [m_1 - \alpha_{1t} - (v\alpha_0)_y - (w\alpha_0)_z]\,(-e^{\xi}) - u_x \xi e^{\xi} - \frac{\partial^2 \Psi_1}{\partial \xi^2} = 0, \\ &\Psi_1(-\infty, \eta, \zeta, \tau) = 0, \quad \Psi_1(0, \eta, \zeta, \tau) = 0, \quad [\partial \Psi_1/\partial \xi]_{\xi = 0-} = 0. \end{aligned}\right\} \tag{13}$$

This is obtained by using the first two terms of (10) for u and by substituting the lowest-order solution into (5), using $\nabla . \mathbf{v} = 0$. After the Fourier transform with respect to τ has been taken, this equation becomes an ordinary differential equation of the second order. Let tildes denote Fourier transforms and introduce

$$B = \int_0^{\tau} [m_1 - \alpha_{1t} - (v\alpha_0)_y - (w\alpha_0)_z]\,d\tau.$$

Then $\Psi_1 = \Phi + B e^{\xi}$, where

$$i\omega\tilde{\Phi} + \frac{\partial \tilde{\Phi}}{\partial \xi} - \frac{\partial^2 \tilde{\Phi}}{\partial \xi^2} = \tilde{u}_x \xi e^{\xi}, \tag{14}$$

ω being the transform variable. The general solution to (14) is

$$\begin{aligned} \tilde{\Phi} = {} & \exp(\lambda_1 \xi)\left(\tilde{A}_1 - \frac{\tilde{u}_x}{\lambda_1 - \lambda_2}\int_0^{\xi} \xi \exp[(1-\lambda_1)\xi]\,d\xi\right) \\ & + \exp(\lambda_2 \xi)\left(\tilde{A}_2 + \frac{\tilde{u}_x}{\lambda_1 - \lambda_2}\int_0^{\xi} \xi \exp[(1-\lambda_2)\xi]\,d\xi\right), \end{aligned} \tag{15}$$

where $\lambda_{1,2} = \tfrac{1}{2}[1 \pm (1 + 4i\omega)^{\frac{1}{2}}]$ are the characteristic values of the homogeneous equation, with $\operatorname{Re}\lambda_1 > 0$, $\operatorname{Re}\lambda_2 < 0$, $\lambda_1 + \lambda_2 = 1$ and $\lambda_1 \lambda_2 = -i\omega$. The upstream boundary condition requires

$$\tilde{A}_2 = -[\tilde{u}_x/(\lambda_1 - \lambda_2)]\int_0^{-\infty} \xi \exp[(1-\lambda_2)\xi]\,d\xi,$$

and the boundary conditions at $\xi = 0$ give

$$\tilde{A}_1 + \tilde{A}_2 = -\tilde{B} \quad \text{and} \quad \tilde{A}_1\lambda_1 + \tilde{A}_2\lambda_2 = -\tilde{B}.$$

These last three relationships imply that $\tilde{u}_x = \tilde{B}_\tau \lambda_1$, which may be used along with $\overline{u} = 0, \overline{\alpha} = 0$ and the assumption that the problem is one-dimensional, in the sense that transverse derivatives of time averages equal zero, to show that

$$m_1 = 0, \quad \tilde{\alpha}_{1t} = -[\tilde{u}_x/\lambda_1 + (\widetilde{v\alpha_0})_y + (\widetilde{w\alpha_0})_z] \tag{16}$$

and

$$\tilde{\Psi}_1 = -\frac{\tilde{u}_x}{i\omega}\left[\frac{1-\exp(-\lambda_2\xi)}{\lambda_2} - \xi\right] e^{\xi}. \tag{17}$$

From (17) and the characteristic value λ_2 it is easily shown that the zero-frequency value of Ψ_1 is zero. Therefore the expansion to first order in ϵ provides no correction to the mean profile in the moving frame of reference. Moreover, $m_1 = 0$ states that to this order there is no difference between turbulent and laminar flame speeds. For these reasons we must study the second-order perturbation, but to determine the second-order correction to the turbulent velocity u_T, we need to consider only the time average of the second-order outer solution. The differential equation to be investigated is therefore

$$\overline{\frac{\partial \Psi_2}{\partial \xi}} + \overline{B_\tau \frac{\partial \Psi_1}{\partial \xi}} + \overline{u_x \xi \frac{\partial \Psi_1}{\partial \xi}} + \overline{v \frac{\partial \Psi_1}{\partial \eta}} + \overline{w \frac{\partial \Psi_1}{\partial \zeta}} = \overline{\frac{\partial^2 \Psi_2}{\partial \xi^2}} + [\overline{|\nabla \alpha_0|^2} - m_2](-e^{\xi}), \tag{18}$$

with boundary conditions from the second-order terms of the gradient expansions of the full boundary conditions. From (9) and the equations following that expression it is easily seen that the boundary conditions for (18) are

$$[\partial \Psi_2/\partial \xi]_{\xi=0-} = \tfrac{1}{2}|\nabla \alpha_0|^2, \quad \Psi_2(0,\eta,\zeta,\tau) = 0, \quad \Psi_2(-\infty,\eta,\zeta,\tau) = 0. \tag{19}$$

By integration of (18) from $\xi = -\infty$ to $\xi = 0$, we find that

$$m_2 = \tfrac{1}{2}\overline{|\nabla \alpha_0|^2}, \tag{20}$$

where use has been made of results obtained at lower orders and of the boundary conditions. Use has also been made of the facts that $\nabla . \mathbf{v} = 0$ and that all of the transverse derivatives of any time average are zero.

4. Flame speeds

The results of this analysis give the influence of large-scale turbulence on the flame speed and the flame structure. The flame-speed correction is the easiest to understand. From $u_T/u_L = m$, $m_0 = 1$, $m_1 = 0$ and (20) it is seen that $u_T = u_L(1 + \frac{1}{2}\overline{|\nabla \alpha_0|^2})$ in the first non-trivial approximation. Since $\alpha_{0t} = u$, if for simplicity of presentation we select an initial time such that $\alpha_0 = 0$ at $t = 0$, then in terms of the longitudinal component u of the turbulent fluctuation in velocity, the flame-speed formula

$$u_T = u_L\left(1 + \tfrac{1}{2}\overline{\left[\frac{\partial}{\partial y}\int_0^t u\,dt\right]^2} + \tfrac{1}{2}\overline{\left[\frac{\partial}{\partial z}\int_0^t u\,dt\right]^2}\right) \tag{21}$$

is obtained. It may be noted that, although u, t, y and z have been defined as non-dimensional quantities, this scaling cancels in (21), so that the symbols may be viewed as dimensional. The result in (21) shows that only the mean-square value of the transverse gradient of the Eulerian displacement

$$a(x,y,z,t) = \int_0^t u(x,y,z,t)\,dt$$

influences the flame speed in the first approximation. This is readily understood from a wrinkled-laminar-flame model.

To fix ideas, first assume that there is no z dependence of the displacement a. If an element of the laminar-flame front of length dy in the transverse direction experiences a differential displacement of its end points by an amount da, then by simple geometry the length of the element of the flame front is $dl = dy/\cos\theta$, where $\tan\theta = da/dy$. In this case dl/dy is the ratio of an element of flame area dA_f to an element of cross-sectional area dA of the flow. For small angles θ, $1/\cos\theta \simeq 1/(1-\frac{1}{2}\theta^2) \simeq 1+\frac{1}{2}\theta^2 \simeq 1+\frac{1}{2}\tan^2\theta$. Therefore $dA_f/dA = 1+\frac{1}{2}(da/dy)^2$ to a first approximation. If there is a z dependence of the displacement a as well, then similarly $dA_f/dA = 1+\frac{1}{2}(\partial a/\partial y)^2+\frac{1}{2}(\partial a/\partial z)^2$. The average of this expression provides the average ratio of the wrinkled-flame area to the cross-sectional area: $\overline{(A_f/A)} = 1+\frac{1}{2}(\overline{a_y^2}+\overline{a_z^2})$. In the wrinkled-flame model, the ratio of the turbulent to the laminar flame speed is simply this average area ratio. Therefore it is seen that the result obtained by elementary mechanistic reasoning concerning wrinkled flames is entirely in agreement with (21). It may be inferred that, to the order to which the analysis has been carried, effects of neither strain rate nor flame curvature on the laminar flame speed influence the turbulent flame speed.

Equation (21) differs from formulae previously given for the turbulent flame speed. A popular formula of wrinkled-flame theory, $u_T = u_L + (\overline{\hat{u}^2})^{\frac{1}{2}}$ (e.g. Williams 1965), differs functionally from (21) in that it involves u linearly rather than quadratically. Those formulae which have been proposed which involve u quadratically do not contain its time integral, an operation essential for the extraction of the displacement that influences the extent of wrinkling. If an extended Taylor hypothesis is introduced to relate dt to dx, then (21) becomes

$$u_T = u_L + \left[\overline{\left(\int_0^x \hat{u}_y\,dx\right)^2} + \overline{\left(\int_0^x \hat{u}_z\,dx\right)^2}\right]\Big/(2u_L),$$

which is like a formula of Shelkin (Williams 1965) if the x integral 'cancels' the y and z derivatives, as it does, for example, if the fluctuating velocity field is isotropic. The result that leads to (21) resembles closely a formula given by Kuznetsov (1975), the present analysis corresponding to the limit of that formula in which the wrinkled laminar flame always propagates normal to itself at the constant velocity u_L.

If the integral were absent in (21), then the ratio of the mean-square intensity of longitudinal velocity fluctuations to the square of the transverse Taylor scale would appear in the flame-speed formula. Because of the integral, a Taylor-like scale for the fluctuating longitudinal displacement appears instead. For axisymmetric turbulence, $\overline{a_y^2} = \overline{a_z^2}$ and (21) can be written as $u_T = u_L(1+\overline{a_y^2})$. If the turbulence is isotropic and if a Taylor hypothesis is introduced, then $\overline{a_y^2} = u_1^2/u_L^2$, where $u_1 = (\overline{\hat{u}^2})^{\frac{1}{2}}$ denotes the root-mean-square velocity fluctuation in the streamwise direction. In this case, the simple formula $u_T = u_L + u_1^2/u_L$ is obtained. This expression differs from the expansion of the Shelkin formula for small u_1/u_L only in the absence of a factor of 2 multiplying u_1^2/u_L,

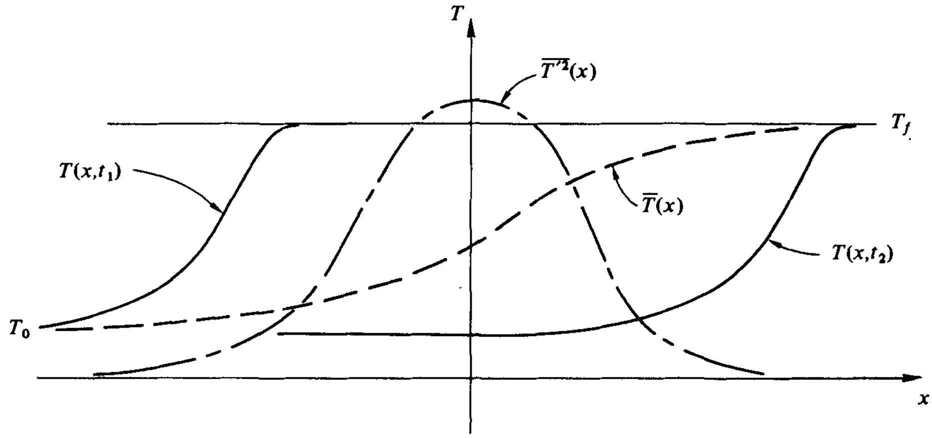

FIGURE 1. Schematic diagram of turbulent flame structure.

but it agrees precisely with equation (24) of Williams (1970). From this result it may be noted that even if the root-mean-square Eulerian displacement varies with time owing to non-stationarity of a, the turbulent flame speed u_T still remains constant as a consequence of stationarity of **v**. In effect, the transverse spatial derivative in (21) removes growth in time.

From the simple formula $u_T = u_L + u_1^2/u_L$ it is seen that u_T/u_L is independent of the turbulence scale, at a fixed relative intensity u_1/u_L, in the limit of large-scale turbulence. More generally, let $\frac{1}{2}(\overline{a_y^2} + \overline{a_z^2}) = (u_1 T_1/l)^2$, where l is the transverse Taylor-like scale for longitudinal displacement and T_1 is a time characteristic of the integral of $\hat{u}$. With $l_1 = u_L T_1$ a length characteristic of the streamwise integral of $\hat{u}$, the same result is obtained if the ratio l_1/l is independent of the scale. There is some experimental evidence favouring the absence of a dependence of u_T/u_L on the turbulence scale in large-scale turbulence (Ballal & Lefebvre 1974), although most of the experimental work has been done at turbulence intensities that are high enough to make the applicability of the present analysis uncertain, since when l_1/l is of order unity or greater the small-gradient expansion requires small velocity fluctuations ($u_1/u_L \ll 1$). Specifically, it may be stated that if the fluctuating displacement field is isotropic, and if a Taylor hypothesis is applicable thereto, then the present theory is restricted to turbulence of intensity sufficiently low for u_1/u_L to be small. In general, this restriction is not present, since only the gradients of the Eulerian displacement need to be small.

5. Flame structure

The dominant aspects of the flame structure may be understood most easily by reference to figure 1. The curves labelled $T(x, t_1)$ and $T(x, t_2)$ represent the laminar flame structure at two different times that correspond closely to two extreme values of the displacement of the laminar flame. The advantage of adopting the moving co-ordinate in (5) is that the subsequent analysis automatically accounts for this displacement. In the first approximation, the main factor affecting the turbulent flame structure is this displacement. While the result $\alpha_{0t} = u$, obtained after (12), implies that in the first approximation the flame-front displacement equals the Eulerian displacement of the upstream mixture, the correction in (16) shows that to first order ϵ there are differences between the instantaneous motion of the flame and that of the fluid. The first approximation to the mean profile $\overline{T}(x)$ is obtained by averaging the instantaneous laminar flame profiles over all time. The profile of the average intensity of temperature fluctuations $\overline{T'^2}(x)$ also has a contribution in a first approximation from a suitable average of the moving laminar flame. The analysis in the second part of § 3 is needed only for obtaining corrections to these results, as given in (17).

To calculate profiles of various averaged quantities through the flame begin with $Y(x,y,z,t) = \Psi_L(x-\alpha)+\Psi_1(x-\alpha,y,z,t)\dots$. From this it is seen that, even in the first approximation, averages of any function of Y involve the complete probability density function of $\alpha_0 = a(0,y,z,t)$. In the second approximation, joint probability density functions for a, u_x, etc., enter through (17). The outer solution for Ψ_L may be used to show that at lowest order the average temperature profile is determined by

$$\overline{Y}(x) = \int_{-\infty}^{\infty} [1-\exp(x-\alpha_0)]\, H(-x+\alpha_0)\, P(\alpha_0)\, d\alpha_0,$$

where $P(\alpha_0)$ is the probability density function for α_0.

In general, to perform an explicit calculation of $\overline{Y}(x)$ an expansion of $P(\alpha_0)$ about a Gaussian form may be selected. For illustrative purposes, assume that $P(\alpha_0)$ is precisely Gaussian with zero mean: $P(\alpha_0) = (2\pi\delta^2)^{-\frac{1}{2}}\exp[-\alpha_0^2/(2\delta^2)]$, where $\delta = (\overline{\alpha_0^2})^{\frac{1}{2}}$. Then

$$\overline{Y}(x) = \frac{1}{2}\left[\operatorname{erfc}\left(\frac{x}{2^{\frac{1}{2}}\delta}\right) - \exp\left(x+\frac{\delta^2}{2}\right)\operatorname{erfc}\left(\frac{x}{2^{\frac{1}{2}}\delta}+\frac{\delta}{2^{\frac{1}{2}}}\right)\right], \tag{22}$$

which approaches the laminar profile as $\delta \to 0$ and $\frac{1}{2}\operatorname{erfc}(x/2^{\frac{1}{2}}\delta)$ as δ becomes large. Curves of $\overline{Y}(x)$ for various values of δ, according to (22), are shown in figure 2. Note that the characteristic thickness of the turbulent flame for sufficiently large values of δ then becomes $x_T \approx \delta d = l_1 u_1/u_L$, if the previously written expression is employed to estimate the displacement in terms of the intensity and a characteristic longitudinal scale of the turbulent displacement field. The resultant linear dependence of the flame thickness on a turbulence scale and on the root-mean-square turbulent velocity fluctuation seems reasonable. It may be observed that, if δd is larger than the flame thickness, in the mean the turbulent flame is not divided into convective–diffusive and reactive–diffusive zones but instead has some reaction occurring nearly everywhere. Note also that turbulence scales may influence the flame thickness without affecting the flame speed in the large-scale limit.

One aspect of the result for the flame thickness deserves further emphasis. Observe that, roughly, $\delta = (\overline{a^2})^{\frac{1}{2}}$. Stationarity of $\mathbf{v}$ does not imply stationarity of the time integral a. The Lagrangian displacement of fluid elements by turbulent fluctuations is known to be non-stationary, constituting the process of turbulent diffusion. Specifically, a linear dependence on time is ascribed to the Lagrangian version of $\overline{a^2}$ at large times, as a consequence of the non-vanishing of the time integral (0 to ∞) of the autocorrelation of the Lagrangian velocity. Whether the Eulerian displacement a is stationary for grid turbulence remains an unresolved question of fundamental importance from both the theoretical and the experimental viewpoint (Comte-Bellot & Corrsin 1971). If a is stationary, then the thickness of the turbulent flame is constant, independent of time. However, if the variance $\overline{a^2}$ grows with time, then the turbulent

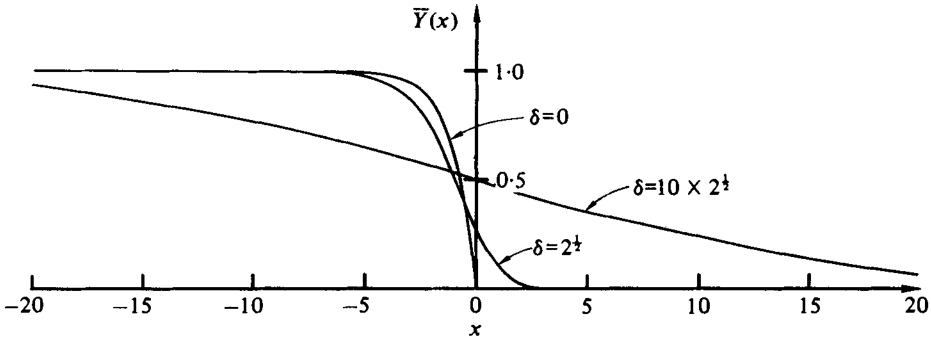

FIGURE 2. Profiles of mean reactant concentration, according to (22).

flame thickness also grows, by a type of turbulent–diffusion process, even though the turbulent flame speed remains constant. Kuznetsov (1975) mentioned this effect and offered some rationalization of it. The present results suggest that experiments on turbulent flames might help to clarify this fundamental problem in grid turbulence.

If a is non-stationary, there are implications concerning the validity of the present analysis. If $\overline{\alpha^2}$ grows with time then a gradient expansion, which effectively requires $(\alpha u_x)^2 \ll u^2$, is restricted in time. If $\hat{u}_x^2 \sim \hat{u}^2/\lambda_1^2$, where λ_1 is a Taylor scale, then $\overline{\hat{a}^2} \ll \lambda_1^2$ is obtained in a first approximation. By analogy with the Lagrangian displacement, $\overline{\hat{a}^2} \sim \hat{t}\,\overline{\hat{u}^2} L_1/u_T$, where L_1 is a longitudinal integral length scale for the turbulence, and the restriction on time becomes $L_1/u_T \ll \hat{t} \ll (L_1/u_T)\,(\lambda_1/L_1)^2\,(\overline{\hat{u}^2}/u_T^2)^{-1}$, the first inequality being necessary for use of the given formula for $\overline{\hat{a}^2}$. Since usual estimates place $\lambda_1/L_1 \lesssim 1$, it is seen that with these ideas a low turbulence intensity is implied by the gradient expansion. On the other hand, if a is stationary then the restriction on time does not arise.

With $Y' = \bar{Y}$, it is easy to calculate a profile of $\overline{Y'^2}$ in a manner similar to that employed in deriving (22). The result will not be written here because the formula is too long. Qualitatively, the profile is like that illustrated in figure 1.

6. Turbulent transport

A quantity of considerable interest in modelling approximations for turbulent flow is the streamwise turbulent transport $\overline{uY'}$. In the first approximation, this quantity is $\overline{uY'} = \overline{u(Y-\bar{Y})} = \overline{uY} = \overline{u\Psi_L(x-\alpha_0)}$, the average to be taken keeping x fixed. Since $u = \alpha_{0t}$, we may write

$$\overline{u\Psi_L(x-\alpha_0)} = \frac{\partial}{\partial t}\overline{\int_0^{x-\alpha_0} \Psi_L(\xi)\,d\xi} = \frac{\partial}{\partial t}\overline{\{(\alpha_0-x)+[\exp(x-\alpha_0)-1]\}H(\alpha_0-x)}, \qquad (23)$$

where the outer solution for Y_L has been employed in obtaining the second equality. Although further reduction of this expression would require use of the probability density function $P(\alpha_0)$, it appears that generally $\overline{u\Psi_L(x-\alpha_0)}$, as given by this formula, will be non-negative. Note, for example, that the quantity to be averaged is an increasing function of α_0 and that for small α_0 an expansion yields

$$\overline{u\Psi_L(x-\alpha_0)} = \tfrac{1}{2}(\overline{\alpha_0^2})_t\, e^x H(-x),$$

which is non-negative and independent of t if $\overline{\alpha_0^2}$ increases linearly with t. This result could be consistent with a gradient approximation to turbulent transport,

$$\overline{uY'} = -D\,d\overline{Y}/dx,$$

where D is an Eulerian turbulent diffusion coefficient. For small α_0 we find $D = \tfrac{1}{2}\overline{(\alpha_0^2)_t}$, which according to conventional ideas is proportional to the intensity u_1^2 and the longitudinal integral scale L_1 of the longitudinal turbulent velocity fluctuations (dimensionally, $\hat{D} \sim u_1^2 L_1/u_L$).

However, $\overline{u\Psi_L(x-\alpha_0)} = 0$ if α_0 is stationary, since the identity $u = \alpha_{0t}$ implies that the quantity to be averaged is the time derivative of a function of α_0. Therefore, in this case the first non-zero contribution to $\overline{uY'}$ arises at the next order in the gradient expansion. Care must be taken in developing this expansion in the laboratory frame because the rapid change in the derivative of $\Psi_L(x-\alpha)$ in the inner zone prevents a Taylor expansion about $x-\alpha_0$ from being carried further than the leading term $\Psi_L(x-\alpha_0)$ unless α_1 is of order β^{-1} or smaller. To first order in ϵ, the velocity that appears in $\overline{uY'}$ is $u(0,y,z,t)+xu_x(0,y,z,t)$, so that

$$\overline{uY'} = \overline{u\Psi_L(x-\alpha)} + \overline{u_x x\Psi_L(x-\alpha)} + \overline{u\Psi_1(x-\alpha,y,z,t)} + \ldots,$$

where, consistent with the earlier definitions (10), on the right-hand side u and u_x are evaluated at $x = 0$. Since $\alpha_{0t} = u$, the expansion $\alpha_t = \alpha_{0t} + \alpha_{1t} + \ldots$ gives

$$u = \alpha_t - \alpha_{1t} + \ldots,$$

whence $\overline{u\Psi_L(x-\alpha)} = \overline{\alpha_t\Psi_L(x-\alpha)} - \overline{\alpha_{1t}\Psi_L(x-\alpha)} + \ldots$, the first term of which vanishes if α is a stationary random variable. Therefore

$$\overline{uY'} = \overline{-\alpha_{1t}\Psi_L(x-\alpha)} + \overline{u_x x\Psi_L(x-\alpha)} + \overline{u\Psi_1(x-\alpha,y,z,t)} + \ldots, \tag{24}$$

where α_{1t} and Ψ_1 are given in (16) and (17). Further reduction of (24) requires use of approximations concerning α_{1t} and Ψ_1.

Considering the low-frequency limit appropriate for large-scale turbulence, we may put $\lambda_1 \approx 1+i\omega$ and $\lambda_2 \approx -i\omega$ in (16) and (17), so that $\alpha_{1t} \approx -[u_x + (v\alpha_0)_y + (w\alpha_0)_z]$ and $\Psi_1 \approx -u_x \tfrac{1}{2}(x-\alpha)^2 e^{x-\alpha}$, giving to lowest order in ϵ

$$\begin{aligned}\overline{uY'} = {} & \overline{-u_x x \exp(x-\alpha_0) H(\alpha_0 - x)} - \overline{[u_x + (v\alpha_0)_y + (w\alpha_0)_z] \exp(x-\alpha_0) H(\alpha_0 - x)} \\ & - \overline{(u^2)_x (x-\alpha_0)^2 \exp(x-\alpha_0) H(\alpha_0 - x)},\end{aligned} \tag{25}$$

where $\overline{u}_x = 0$ and homogeneity in y and z have been used in averages. Knowledge of the joint probability density function for u_x and α_0 is needed for calculating the average in (25). A feeling for some characteristics of the result can be obtained from an expansion for small α_0, although such an expansion can be compatible with the low-frequency limit only for very low turbulence intensity. Specifically, if t_T and t_L are characteristic times for turbulence and laminar flame transit, respectively, then since

$$\alpha_0 \approx (\hat{u}/u_L)(t_T/t_L),$$

$\alpha_0 \ll 1$ requires $\hat{u}/u_L \ll t_L/t_T$, while in addition $t_L/t_T \ll 1$ in the low-frequency limit. The first non-vanishing terms in the expansion of (25) in powers of α_0 are

$$\overline{u_x\alpha_0}(1+x)\,e^x H(-x) - \overline{(u^2)_x}\,x^2 e^x H(-x),$$

but since $u \approx \alpha_0 t_L/t_T$, we have $u \ll \alpha_0$ for $t_L \ll t_T$, which shows that the second of these terms is small compared with the first in the current approximation. Thus

$$\overline{uY'} = \overline{(u_x \alpha_0)}\,(1+x)\,e^x H(-x) \tag{26}$$

is valid for slow and very weak turbulence, with a large scale, when the flame-front displacement α (in first approximation the Eulerian displacement a) is a stationary random variable.

In (26) it is seen that $\overline{uY'}$ changes sign at $x = -1$ and drops sharply to zero at $x = 0$. Note therefore that in this limiting case any gradient approximation $\overline{uY'} = -Dd\overline{Y}/dx$ to the turbulent transport would have to employ a turbulent diffusion coefficient D that changes sign at $x = -1$. Since negative coefficients of diffusion are undesirable, it may be concluded that this particular conventional approximation for closure in modelling of turbulent reacting flows is inapplicable in the limit considered. In general, gradient approximations for turbulent transport of reacting species in flames do not have sufficient fundamental justification and must be considered to be of doubtful validity.

7. Concluding remarks

An alternative approach to the problem analysed herein is to use a co-ordinate system that moves in the longitudinal direction at the fluctuating velocity u of the fluid, instead of at the velocity of the reactive–diffusive zone. Although we have completed the entire theory by this alternative approach, none of the analysis is included herein because it involves excessively tedious algebra. In particular, solutions up to second order in β^{-1} in the reactive–diffusive zone must be developed as a consequence of the displacement of this zone with respect to the adopted co-ordinates. Contrary to the present method, the alternative approach involves performing the expansion in ϵ prior to that in β^{-1}. Since we found that the two approaches lead to the same results, it may be inferred that, subject to certain ordering restrictions that arise in the analysis of the reactive–diffusive zone by the alternative approach, the order in which the expansions are performed is irrelevant. Our experience in pursuing alternative paths has shown that if a reactive–diffusive zone exists then appreciable simplification occurs when a co-ordinate system is selected that moves with this zone.

In the present work attention is focused only on the simplest effects produced by the spatial variation of the fluctuating velocity field. New phenomena occur when the Lewis number of the limiting component is different from one or in the case of a two-reactant system near stoichiometric composition. These problems can also be treated by the method of gradient expansions. In the stable case, the main influence of a Lewis number different from unity is that flame-sheet wrinklings produced by the fluctuating velocity field trigger fluctuation of the combustion temperature. Since the local flame velocity is sensitive to the combustion temperature, new effects are added to those described herein. The results obtained for a Lewis number differing from unity by an amount of the order of magnitude of β^{-1} will be presented in a following paper.

It is worthwhile to note that the method of gradient expansions, used throughout this work, also can be employed to solve other kinds of problem. One example is the self-evolution of an initially perturbed flame front in a quiescent gas. The corresponding evolution equation, obtained as above for a Lewis number of unity, is

$$\alpha_t = -\tfrac{1}{2}|\nabla\alpha|^2 + \nabla^2\alpha + \dots,$$

up to terms of second order in gradients. Notice that here the first term corresponds to normal propagation at the planar flame speed; the second term is a curvature effect similar to that discussed by Markstein (1964). Owing to the time average, this last effect disappears in the turbulent wrinkled flame. This evolution equation agrees, up to second order in gradients, with that obtained by Sivashinsky (1977*a*, *b*). It is worthwhile to notice that in this equation the self-evolution of the flame front is of second order in gradients, while (16) shows that the spatial dependence of the upstream field of fluctuating velocity forces an evolution of the flame front at first order in gradients. Thus, on a time-resolved basis, the forced evolution exerts the larger influence on flame motion.

A second example of the use of the present method is analysis of the stationary shape of a Bunsen flame. The approach yields the equation

$$\tfrac{1}{2}|\nabla\alpha|^2 = (u - u_L)/u_L + \nabla^2\alpha,$$

where u is the velocity of the fresh mixture. In two dimensions, the solution is $\alpha_y = -n \tanh(\tfrac{1}{2}ny)$, where $n = [2(u-u_L)/u_L]^{\frac{1}{2}}$. This type of result has been developed by Sivashinsky (see references quoted by Sivashinsky 1977*a*) through a somewhat different expansion.

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REFERENCES

Ballal, D. R. & Lefebvre, A. H. 1974 Turbulence effects on enclosed flames. *Acta Astronautica* **1**, 471–483.

Comte-Bellot, G. & Corrsin, S. 1971 Simple Eulerian time correlation and narrow-band velocity signals in grid-generated, 'isotropic' turbulence. *J. Fluid Mech.* **48**, 273–337.

Joulin, G. & Clavin, P. 1978 Stability analysis of nonadiabatic flame. Submitted to *Combust. Flame.*

Kuznetsov, V. R. 1975 Certain peculiarities of movement of a flame front in a turbulent flow of homogeneous fuel mixtures. *Fiz. Goreniya Vzryva* **11**, 574–581.

Landau, L. 1944 Theory of slow combustion. *Acta Physicochima. USSR* **19**, 77–85.

Libby, P. A. & Williams, F. A. 1976 Turbulent flows involving chemical reactions. *Ann. Rev. Fluid Mech.* **8**, 351–376.

Markstein, G. H. 1964 *Nonsteady Flame Propagation. AGARDograph* no. 75.

Sivashinsky, G. I. 1977*a* Diffusional–thermal theory of cellular flames. *Combust. Sci. Tech.* **15**, 137–145.

Sivashinsky, G. I. 1977*b* Nonlinear analysis of hydrodynamic instability in laminar flames. *Acta Astronautica* **4**, 1177–1206.

Williams, F. A. 1965 *Combustion Theory.* Addison-Wesley.

Williams, F. A. 1970 An approach to turbulent flame theory, *J. Fluid Mech.* **40**, 401–421.

Williams, F. A. 1971 Theory of combustion in laminar flows. *Ann. Rev. Fluid Mech.* **3**, 171–188.

Williams, F. A. 1975 A review of some theoretical considerations of turbulent flame structure. *Analytical and Numerical Methods for Investigation of Flow Fields with Chemical Reactions, Especially Related to Combustion. AGARD Conf. Proc.* no. 164, pp. III-1–III-25.